



## Cambridge International AS & A Level

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**MATHEMATICS**

**9709/13**

Paper 1 Pure Mathematics 1

**October/November 2020**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

1 (a) Express  $x^2 + 6x + 5$  in the form  $(x + a)^2 + b$ , where  $a$  and  $b$  are constants. [2]

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(b) The curve with equation  $y = x^2$  is transformed to the curve with equation  $y = x^2 + 6x + 5$ . Describe fully the transformation(s) involved. [2]

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2 The function  $f$  is defined by  $f(x) = \frac{2}{(x+2)^2}$  for  $x > -2$ .

(a) Find  $\int_1^{\infty} f(x) \, dx$ . [3]

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(b) The equation of a curve is such that  $\frac{dy}{dx} = f(x)$ . It is given that the point  $(-1, -1)$  lies on the curve.

Find the equation of the curve. [2]

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- 3 Solve the equation  $3 \tan^2 \theta + 1 = \frac{2}{\tan^2 \theta}$  for  $0^\circ < \theta < 180^\circ$ . [5]

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- 4 A curve has equation  $y = 3x^2 - 4x + 4$  and a straight line has equation  $y = mx + m - 1$ , where  $m$  is a constant.

Find the set of values of  $m$  for which the curve and the line have two distinct points of intersection. [5]

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- 5 In the expansion of  $(a + bx)^7$ , where  $a$  and  $b$  are non-zero constants, the coefficients of  $x$ ,  $x^2$  and  $x^4$  are the first, second and third terms respectively of a geometric progression.

Find the value of  $\frac{a}{b}$ .

[5]

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6 The function  $f$  is defined by  $f(x) = \frac{2x}{3x-1}$  for  $x > \frac{1}{3}$ .

(a) Find an expression for  $f^{-1}(x)$ . [3]

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(b) Show that  $\frac{2}{3} + \frac{2}{3(3x-1)}$  can be expressed as  $\frac{2x}{3x-1}$ . [2]

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(c) State the range of  $f$ . [1]

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- 7 The first and second terms of an arithmetic progression are  $\frac{1}{\cos^2 \theta}$  and  $-\frac{\tan^2 \theta}{\cos^2 \theta}$ , respectively, where  $0 < \theta < \frac{1}{2}\pi$ .

(a) Show that the common difference is  $-\frac{1}{\cos^4 \theta}$ . [4]

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(b) Find the exact value of the 13th term when  $\theta = \frac{1}{6}\pi$ . [3]

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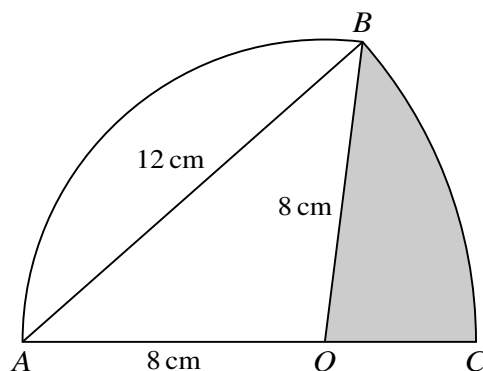
8 The equation of a curve is  $y = 2x + 1 + \frac{1}{2x + 1}$  for  $x > -\frac{1}{2}$ .

(a) Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . [3]

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(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

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In the diagram, arc  $AB$  is part of a circle with centre  $O$  and radius  $8$  cm. Arc  $BC$  is part of a circle with centre  $A$  and radius  $12$  cm, where  $AOC$  is a straight line.

(a) Find angle  $BAO$  in radians.

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(b) Find the area of the shaded region.

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(c) Find the perimeter of the shaded region.

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10 A curve has equation  $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$  where  $x > 0$  and  $k$  is a positive constant.

(a) It is given that when  $x = \frac{1}{4}$ , the gradient of the curve is 3.

Find the value of  $k$ .

[4]

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(b) It is given instead that  $\int_{\frac{1}{4}k^2}^{k^2} \left( \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2} \right) dx = \frac{13}{12}$ .

Find the value of  $k$ .

[5]

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11 A circle with centre  $C$  has equation  $(x - 8)^2 + (y - 4)^2 = 100$ .

(a) Show that the point  $T(-6, 6)$  is outside the circle. [3]

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Two tangents from  $T$  to the circle are drawn.

(b) Show that the angle between one of the tangents and  $CT$  is exactly  $45^\circ$ . [2]

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The two tangents touch the circle at  $A$  and  $B$ .

- (c) Find the equation of the line  $AB$ , giving your answer in the form  $y = mx + c$ . [4]

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- (d) Find the  $x$ -coordinates of  $A$  and  $B$ . [3]

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